You are given a string num, representing a large integer. Return *the****largest-valued odd****integer (as a string) that is a****non-empty substring****of*num*, or an empty string*""*if no odd integer exists*.

A **substring** is a contiguous sequence of characters within a string.

**Example 1:**

**Input:** num = "52"

**Output:** "5"

**Explanation:** The only non-empty substrings are "5", "2", and "52". "5" is the only odd number.

**Example 2:**

**Input:** num = "4206"

**Output:** ""

**Explanation:** There are no odd numbers in "4206".

**Example 3:**

**Input:** num = "35427"

**Output:** "35427"

**Explanation:** "35427" is already an odd number.

**Constraints:**

* 1 <= num.length <= 105
* num only consists of digits and does not contain any leading zeros.